

# Statistical Design and Analysis III

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# Dose Proportionality

- Dose proportionality may be evaluated in a two-step procedure

(Chow and Liu, Design and Analysis of Bioavailability and Bioequivalence Studies, Marcel Dekker, New York, pp 563–573 (3<sup>rd</sup> ed 2009)

- Let  $\mathbf{y}$  be the response (AUC,  $C_{\max}$ ) and  $\mathbf{x}$  the dose level. Since the standard deviation of  $\mathbf{y}$  increases with the dose, the primary assumption of dose proportionality is that the standard deviation of  $\mathbf{y}$  is proportional to  $\mathbf{x}$ ; that is,

$$\text{Var}(\mathbf{y}) = \mathbf{x}^2\sigma^2,$$

where  $\sigma^2$  consists of inter- and intrasubject variabilities.

# Dose Proportionality

- Two-step procedure
  - Under this assumption, following models are considered to evaluate the relation between response  $Y$  and dose  $x$ :
    - Model 1:  $E(Y | x) = b \cdot x$
    - Model 2:  $E(Y | x) = a + b \cdot x$ , where  $a \neq 0$
    - Model 3:  $E(Y | x) = a \cdot x^b$ , where  $a > 0$  and  $b \neq 0$

# Dose Proportionality

- Two-step procedure
  - Model 1 indicates that the relation between response and dose is linear.  
The dose response curve is a straight line, which passes through the origin.  
This model is commonly referred to as **dose proportionality**.

# Dose Proportionality

- Two-step procedure

- Step 1 (dose proportionality)

All dose dependent parameters (e.g., AUC,  $C_{\max}$ ) are normalized to the dose of the reference prior to comparative analyses.

Multiplicative model as usual in BE.

Following hypotheses are evaluated during statistical analysis (given for bioavailability ratios):

# Dose Proportionality

- Two-step procedure

- Step 1 (dose proportionality)

- $H_{1a0}$ :  $\mu_{\text{test } 1}/\mu_{\text{ref.}} \leq \theta_1$  or  $\mu_{\text{test } 1}/\mu_{\text{ref.}} \geq \theta_2$ : null hypothesis 1a ( $\mu_{\text{test } 1}$  and  $\mu_{\text{ref.}}$  are *not* dose proportional)
    - $H_{1a1}$ :  $\theta_1 < \mu_{\text{test } 1}/\mu_{\text{ref.}} < \theta_2$ : alternative hypothesis 1a ( $\mu_{\text{test } 1}$  and  $\mu_{\text{ref.}}$  are dose proportional)
  - $H_{2a0}$ :  $\mu_{\text{test } 2}/\mu_{\text{ref.}} \leq \theta_1$  or  $\mu_{\text{test } 2}/\mu_{\text{ref.}} \geq \theta_2$ : null hypothesis 2a ( $\mu_{\text{test } 2}$  and  $\mu_{\text{ref.}}$  are *not* dose proportional)
    - $H_{2a1}$ :  $\theta_1 < \mu_{\text{test } 2}/\mu_{\text{ref.}} < \theta_2$ : alternative hypothesis 1a ( $\mu_{\text{test } 2}$  and  $\mu_{\text{ref.}}$  are dose proportional)
  - The interval  $[\theta_1, \theta_2]$  denotes the acceptance range

# Dose Proportionality

- Two-step procedure
  - Step 1 (dose proportionality)
    - If the null hypothesis is rejected for a parameter, dose proportionality is proven within the compared dose levels.
    - If, however, the null hypothesis is not rejected, in a second step **dose linearity** (Model 2), and **departure from dose linearity** (Model 3) has to be evaluated.

# Dose Proportionality

- Two-step procedure

- Step 2 (dose linearity)

Model 2 indicates that the relation between response and the dose follows a straight line with nonzero intercept (**a**). It will be tested using a weighted linear regression with weights equal to  $x^{-1}$  with the original (**untransformed**) data (**x**, **Y**). The hypotheses of primary interest are given as:

- $H_{20}: a=0$  null hypothesis 2 (dose response curve pass through the origin)
    - $H_{21}: a \neq 0$  alternative hypothesis 2 (nonzero intercept)



# Dose Proportionality

- Two-step procedure

- Step 2 (dose linearity)

Model 3 indicates that the relation between response and the dose follows the form of a power curve with the exponent **b**. It will be tested using a weighted nonlinear regression with weights equal to  $x^{-1}$  with the original (untransformed) data (**x**, **Y**).

Model 3 will be evaluated by examining the 95% confidence interval of the exponent **b** for departure from one.

# Dose Proportionality

- Two-step procedure

- Step 2 (dose linearity)

The hypotheses of primary interest are given as:

- $H_{30}: \mathbf{b}=1$  null hypothesis 3 (no departure from dose linearity)

- $H_{31}: \mathbf{b} \neq 1$  alternative hypothesis 3 (dose response curve follows a power curve)

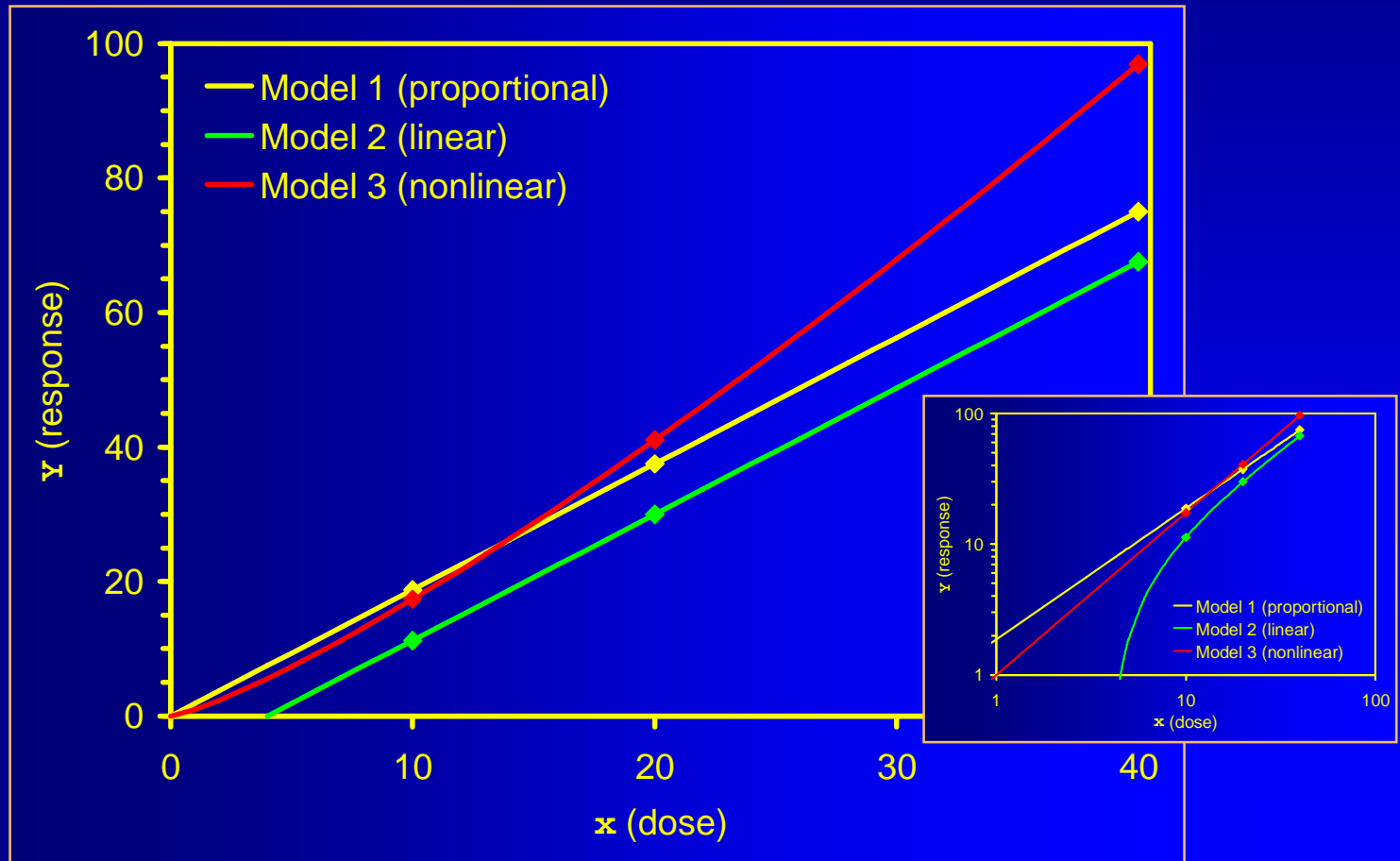
The departure from dose linearity will be evaluated by the 95% confidence interval ( $L, U$ ) for  $\mathbf{b}$  according to following decision criteria:

# Dose Proportionality

- Two-step procedure
  - Step 2 (dose linearity)

if $0.75 < L < 1.0 < 1.25$	no departure from dose linearity ( <i>i.e.</i> , Model 2 holds)
if $1.0 < L < U < 1.25$ or $0.75 < L < U < 1.0$	slight departure from dose linearity, but no practical significance from dose linearity
if $L > 1.25$ or $U < 0.75$	reject hypothesis of dose linearity ( <i>i.e.</i> , Model 3 holds)

# Dose Proportionality



# Dose Proportionality

- Two-step procedure
  - Example (FIM biological, 6 dose levels,  $C_{\max}$ )

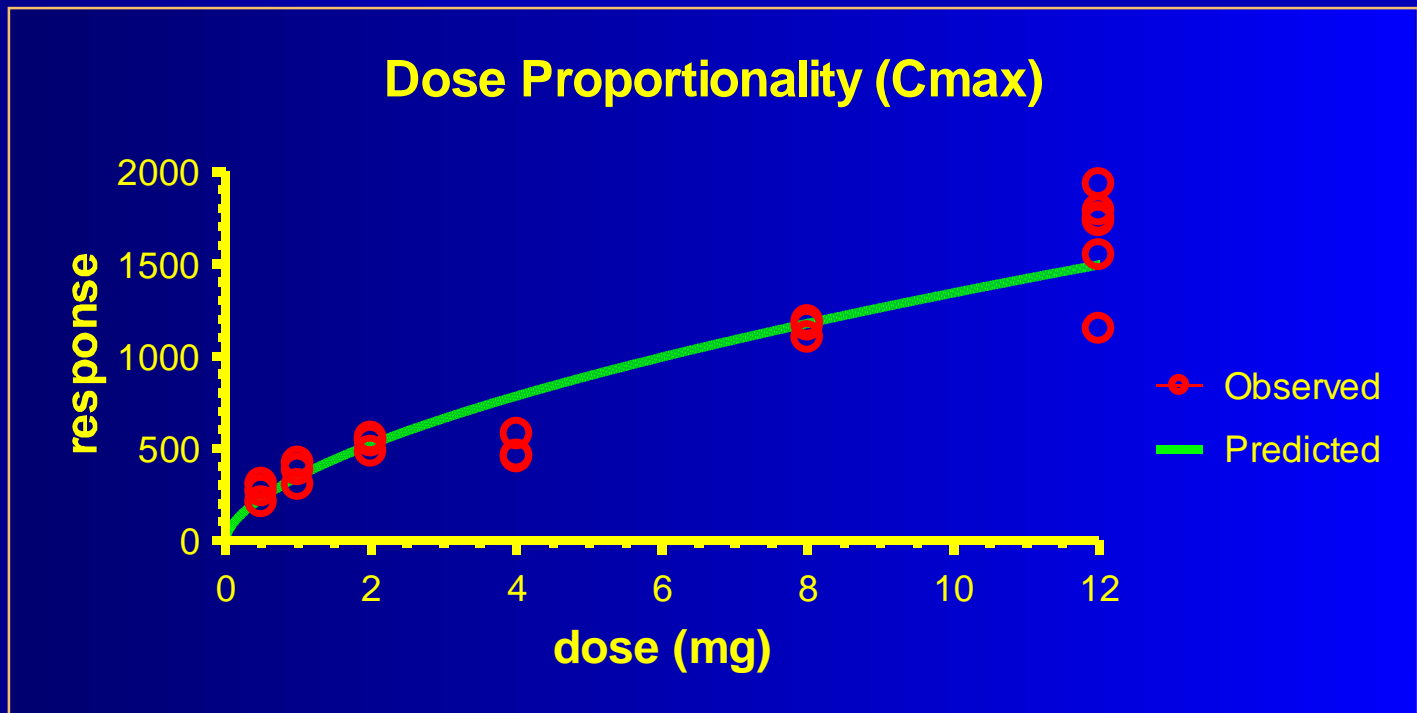
response	<b>b</b>	95 % CI (L,U)		CV%	Corr.
$C_{\max}$	0.587	0.471	0.704	7.28	0.9446

$$U_b < 0.75$$

Model 3 holds (deviation from dose linearity)

# Dose Proportionality

- Two-step procedure
  - Example (FIM, 6 dose levels,  $C_{max}$ )



*Thank You!*

# Statistical Design and Analysis III

*Open Questions?*

*(WinNonlin User model in your handouts  
– use at your own risk!)*

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# Dose Proportionality

## ● WinNonlin user model

```
remark      DOSE PROPORTIONALITY, Chow/Liu 2009 p 564 Models 1-4
remark      data in original (untransformed) scale (X/Y)
remark      weight = 1/x, weights must be provided in column 3
remark      (c) Helmut Schuetz, BEBAC, 1070 Vienna, Austria
model 1
remark      Dose Proportionality
remark      Model 1:  $E(Y)=bx$  (linear through origin)
remark              b1 = slope
remark              weight = 1/x
commands
dnames 'dose' 'response'
npar 1
pname 'b1'
initial 1
nobounds
method 3
weight
end
func 1
f = b1 * x
end
eom
```



# Dose Proportionality

- WinNonlin user model

```
model 2
remark      Dose Proportionality (Dose Linearity)
remark      Model 2:  $E(Y)=a+bx$  (linear)
remark      where  $a \neq 0$ 
remark      a = intercept
remark      b = slope
remark      weight =  $1/x$ 
commands
dnames 'dose' 'response'
npar 2
pname 'a2' 'b2'
initial 0 1
nobounds
method 3
weight
end
func 1
f = a2 + b2 * x
end
eom
```

# Dose Proportionality

- WinNonlin user model

```
model 3
remark      Dose Proportionality (Nonlinear PK)
remark      Model 3:  $E(Y)=a*x^b$  (power function)
remark      where  $a>0$  and  $b\neq 0$ 
remark      a = coefficient
remark      b = exponent ('curvature')
remark      weight =  $1/x$ 
commands
dnames 'dose' 'response'
npar 2
pname 'a3' 'b3'
initial 1 1
nobounds
weight
end
func 1
f = a3 * x ** b3
end
eom
```

# Dose Proportionality

- WinNonlin user model

```
model 4
remark      Dose Proportionality (Nonlinear PK)
remark      Model 4:  $E(Y)=a+c*x^b$  (power function with intercept)
remark      where  $a \neq 0$  and/or  $b \neq 1$ 
remark      a = intercept
remark      b = exponent ('curvature')
remark      c = coefficient
remark      weight = 1/x
commands
dnames 'dose' 'response'
npar 3
pname 'a4' 'b4' 'c4'
initial 0 1 1
nobounds
weight
end
func 1
f = a4 + c4 * x ** b4
end
eom
```